A LANCHESTER BASED MODEL FOR ANALYZING INFANTRY FIRE AND MANEUVER TACTICS

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THESIS

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October 1969

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
October 1969

ABSTRACT

In an effort to analyze small unit fire and maneuver tactics, Lanchester's Square Law is used as the basis for a model relating major combat variables of infantry engagements. An investigation encompassing 90 different computer battle simulations with varying levels of attacking force size, rush distance length, number of units composing the attacking force, and defending force fire distribution is reported. Conclusions of the results of the battle simulations and suggested extensions of the model are discussed

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

$A(t)$	Attacking force size at time (t).
$A1(t)$	The number of maneuvering combatants of attacking force at time t.
$A2_i(t)$	The number of attacking force combatants in the i^{th} unit of the base of fire at time t.
AP	Assault position.
BL	Basic individual load of ammunition (number of rounds carried).
C_A	Attacking force attrition rate coefficient.
C_D	Defending force attrition rate coefficient.
$D(t)$	Defending force size at time t.
$D1(t)$	The number of defending combatants firing on the moving attackers.
$D2(t)$	The number of defending combatants firing on the attacking force base of fire.
$\frac{dA}{dt}$, $\frac{dD}{dt}$	Loss rates of respective forces.
HP	Single shot hit probability, assuming circular normal distribution.
HPA1	Probability that an attacker is killed while running.
$HPA1_1$	Probability that a moving attacker is killed during the TC time units he is prone and getting ready to fire and before another attacking force unit rushes forward.
$HPA2_i$	Probability that an attacker in the base of fire is killed.

HPD	Probability that a defender is killed.
LOD	Line of departure.
p_A, p_D	Combatant's probability of killing opposing forces in individual encounters.
r_A, r_D	Rate at which a combatant encounters the opposing force. (One encounter implies one round fired.)
R, R(t)	Range from attacking force to defending force (meters).
RD	Rush distance; the distance the maneuvering attackers move in a straight line from one prone position to another prone position (meters).
RAD1	Radius of the circular target area equivalent to that presented by an attacker in an up and moving posture (inches).
RAD2	Radius of the circular target area equivalent to that presented by a prone attacker (inches).
RAD3	Radius of the circular target area equivalent to that presented by a defender (inches).
ROA	Rate of advance of attacking force (meters/minute).
ROF	Attacking force rate of fire (rounds/minute).
ROFMAX	Maximum attacking force rate of fire (rounds/minute).
ROFMIN	Minimum attacking force rate of fire (rounds/minute).
t_1	Time it takes the attacking force to come within maximum effective range of the weapons employed.
t_2	Time it takes the attackers to reach the final assault position.
T_{aD}	Time required by defenders to acquire an attacking target.

T_{fD}	Time required by defenders to fire their weapons (i.e., time between trigger pulls).
TAR	Time for the entire attacking force to move one rush distance.
TC	Coordinating time between unit movements of the attacking force.
T_D	Reciprocal of defenders' rate of fire.
TMAXA	Defending force maximum time to acquire an attacking target.
TMAXF	Defending force maximum time to fire.
TMINA	Defending force minimum time to acquire an attacking target.
TMINF	Defending force minimum time to fire.
TR	Time for an attacker to move one rush distance.
u	Total number of units comprising the attacking force.
$\alpha(j)$	Aiming error in mils (radial dispersion of rounds).
$\overline{\alpha}(TR)$	A weighted aiming error used in calculating hit probabilities against the maneuvering combatants.
η	Fraction of attacking force assigned to maneuver.
γ	Fraction of battle time that a combatant is not exposed and, hence, not firing.
γ_{fs}	Fraction of a force not exposing themselves at any instant of time due to fire suppression.
$\gamma_{r\ell}$	Fraction of total battle time used by one combatant to reload.
λ	Fraction of the basic load of ammunition expended by a combatant in the attack.
ρ	Fraction of defending force firing on the moving attackers.

I. SUMMARY

The purpose of the study reported in this thesis was to identify and relate the major variables of small-unit infantry combat engagements in such a manner that different tactical fire and maneuver policies could be studied. With Lanchester's Square Law providing a point of departure, a model of small unit combat was developed in which major parameters of an encounter known to be time or range dependent were so treated, thus incorporating realism of dynamic combat.

The single uncontrolled variable of the model is force size. Force sizes were specified at the start of each computer battle simulation, however these sizes were updated by the computer program every one-tenth of a minute of the battle. The remaining variables were controlled in that they were either assumed, calculated from other data, or direct input values from other research. Values were taken from research by other authors, military recorded statistics, and personal combat knowledge. Success in battle was considered dependent upon infliction of casualties on the opposing force and the range between the forces at the termination of the engagement.

To analyze various tactical combinations and situations a computer program of a small unit engagement was run. The tactics investigated were various constant rush distances for the attacker in combination with the size of the rushing unit. For each battle investigated these two parameters were varied. The rushing unit was taken as a portion of the attacking force, with the entire offensive force moving forward in the attack by incremental rushes. The attackers were opposed by a static-position defending force. The distribution of defensive fire was also varied between battles but remained constant for a given engagement.

The computer output from 90 different battles was analyzed for tactical implications rather than specific numerical answers. This generated battle data implied that rush distances of 30 to 40 meters, in combination with two or three units comprising the attacking force, produced the highest degree of success for the attackers.

The results of the model application support the Marine Corps doctrine of triangular force structure. However, the rush distance results are three to four times greater than those suggested by Marine Corps tactics.

II. INTRODUCTION

"The mission of the rifle squad is to locate, close with, and destroy the enemy by fire and maneuver...." [33] Such a demanding and important mission for small units suggests that considerable support should be given toward the accomplishment of that unit's objectives. Support involves not only military logistic and fire support during combat operations, but also includes effort in planning and preparing for the operation. Research for improved tactical policies, more efficient weapons allocations, better communication and control, and enhancement of ammunition capabilities may produce new concepts which could aid the small unit in successfully completing its mission. Historically, considerable effort in research and modeling has been expended in all these areas except for the area of tactics. Investigation of tactics has been largely empirically oriented with only limited development of formal models to aid tactical decision making. Perhaps the reasons for less emphasis on research and modeling of tactics than on improvement of military equipment was reflected in 1953 by L. F. Richardson. He stated, "Literary people have sometimes wrongly supposed that mathematical expressions can be used to describe the actions of only such objects as follow laws of a rigid mechanical, deterministic type in all particulars." He also cautioned, "Mathematical expressions have, however, their special tendencies to pervert thought: the definiteness may be spurious, existing in the equations but not in the phenomena to be described; and the brevity may be due to the omission of the more important things, simply because they cannot be mathematized." [24]

These concepts expressed by Richardson imply that careful mathematical investigation of a complex subject such as small unit tactics could yield useful results.

Numerous papers have been published on the art of combat modeling; however, they generally address casualty prediction or optimal weapon mixes. Techniques such as Lanchester's equations, Markov processes, and Monte Carlo simulation have been used in modeling combat. A few authors, such as Brackney, Schaeffer, and Deitchman, have written about small unit engagements, but again they seem to approach the situation from the survivability/casualty concept. [6,23,8] The complexities surrounding combatants in small unit engagements and the interweaving of their actions present mathematical challenges which must be tempered by military and analytical judgment; however, probabilistic and mathematical investigations of small unit tactics can at least open unknown doors.

The purpose of this thesis is to identify and relate major variables of small unit infantry combat engagements in such a manner that different tactical fire and maneuver policies can be studied. The thesis proceeds by first reviewing general background knowledge in Lanchester Theories of Combat. Basic tactics are addressed in Chapter IV through explanation of small unit composition, engagement, and maneuver, and by presentation of military definitions. Chapter V includes preliminary exposition of the dynamics of combat engagements and parametric interactions. Then, starting with Lanchester's Square Law, the model is constructed by incorporation of major combat variables into the basic equations. Many of these variables were found to depend on time and range between opposing forces. For those variables, functional expressions are developed and presented in the latter part of Chapter V. The last section of that chapter gives the final form of the fire fight model. In Chapter VI there is an investigation of a small unit in a daylight attack. Additions to the general model assumptions are made followed by an explanation of parameter values which came from published research and military statistics. Results of the computer program used to solve the model are then presented.

Suggested extensions of the model are presented in Chapter VII, together with a discussion of general model conclusions and output trends. An appendix contains the computer program and sample output data for the specific application of the model.

III. LANCHESTER THEORIES OF COMBAT

The dynamics of modern combat have been modeled by F. W. Lanchester [14]. Given a situation where concentration of force is possible, Lanchester formulated the engagement in attrition of opposing forces by two simultaneous differential equations:

$$\frac{dA(t)}{dt} = -C_D \, D(t)$$

and

$$\frac{dD(t)}{dt} = -C_A \, A(t)$$

where A(t) and D(t) are respectively the attacking force and defending force sizes at time t. C_A and C_D are attrition rate coefficients, and represent the constant rate at which an individual combatant kills the opposing force. The state solution of these equations, where force strength is proportional to the square of the number of combatants entering the battle, implies the advantages of force concentration. [17]

Lanchester's Square Law (the above equations) is based on the assumption that combatants attack each other in such a manner that each force may take any enemy unit under fire and may shift fire to another enemy unit when desired. Other assumptions of his model are that each combatant is within weapon range of all enemy units, forces are composed of homogenous units, with possibly different force attrition rate coefficients, location of the enemy is known, and fire distribution is uniform over surviving forces.

Howard Brackney [6] represented attrition rate coefficients as a product of the probabilities of kill in individual encounters (p) and the rates at which opposing combatants encounter each other (r), where an encounter implies one combatant firing one round at an opponent. Using this notation, Lanchester's model of force concentration in combat becomes

$$\frac{dA(t)}{dt} = -p_D r_D D(t) \tag{1}$$

and

$$\frac{dD(t)}{dt} = -p_A r_A A(t) \tag{2}$$

Solution of these equations implies an interesting property of acceleration of the action toward the end of a battle since the last half of a force is annihilated quicker than the first. This is caused by the remaining members of the opposing force being able to shift their fire and concentrate on the enemy's destruction.

The usefulness of direct application of Lanchester's Square Law to small unit combat has been questioned because of the restrictive assumptions of the model. [31] Bonder wrote, "Current literature suggests that the use of Lanchester-type models for prediction of battle results has been hampered by this inability to predict the attrition rate coefficients." [5] It is evident that attrition is dependent on more than the number of firing units and a constant attrition coefficient. Though the basic assumptions of Lanchester are restrictive, it appears that inferences could be made about real combat situations if variable attrition rate coefficients dependent on such battle parameters as range, time, and aiming error could be developed. During the following chapters an attempt shall be made to identify significant battle parameters and to relate them to attrition rate coefficients.

IV. SMALL UNIT TACTICS

Aggressive and decisive action against an enemy to effect total destruction of that enemy force is the underlying substance of offensive combat. Mission, enemy and friendly situations, and terrain, varying from battle to battle and even being dynamic within each engagement, dictate a leader's choice of basic tactics in order to meet changing situations. Hence, this chapter addresses basic definitions and concepts of small unit tactics in preparation for a mathematical development to model the action of combat. Tactics and terminology will be based on U.S. Marine Corps doctrine. [15,33]

In this context a small unit is defined as a unit of fire team, squad, or platoon size. Each fire team consists of four men; each squad is based upon three fire teams; a platoon is formed from three squads. The leaders of squads and platoons are separate from their comprising units. For the purpose of this paper a fire fight designates more than just a firing duel at constant range. The fire fight describes the moving, dynamic combat engagement whereby the offensive small unit attempts to close with and destroy an enemy in static position firing at at the advancing force.

Of the several phases of offensive combat, we shall be primarily concerned with the attack phase. This phase begins when an attacking force crosses a line of departure (LOD). Subsequently, the attackers are forced to deploy and fire upon the enemy in order to move forward. The LOD is an imaginary, easily identifiable line perpendicular to the direction of attack and controlled by friendly forces. Though the final objective of the attacking force is destruction of the enemy, an intermediate objective is movement to an assault position (AP) from which a final assault into the enemy position can be launched.

If after crossing the line of departure the attacking force is fired upon, the unit advances by fire and maneuver. When moving, the attacking force utilizes the cover and concealment provided by the terrain and protection of supporting fires. When the concept of fire and maneuver is employed, subunits and individuals within the attacking force alternate in moving and covering by fire the advance of the others. When applied to a small unit in a frontal attack the term maneuver may be replaced by movement to indicate basic forward movement toward the objective without employment of envelopment tactics. Movement entails an on-line combat formation where individuals are basically abreast of one another. Control of movement of the attacking force is by the unit leader who may move his men by unit rushes or individual rushes. Each rush forward is accompanied by support from the remaining individuals or units which then comprise the base of fire.

Using these foundational definitions and tactical concepts a mathematical model of small unit offensive combat will be developed in the next chapter.

V. FIRE FIGHT MODEL

Underlying any combat engagement is the concept of success. For as many times as there are battles there exist equally many definitions of success. This thesis views success from the position of the attacker. The mission of an attacking force is to destroy the defending force, thus its primary physical objective is to acquire the defenders' position. To accomplish this, movement of the force to an intermediate objective designated as the assault position (AP) is necessary. Hence, the attacking force is to move from a line of departure (LOD) far forward of the enemy to a previously designated AP which is closer to the defended position. It is this movement between the LOD and the AP which is to be modeled, and it is this phase of battle which incorporates offensive fire and maneuver tactics. The model does not address the final assault situation where the entire attacking force rises from the AP, on line, and assaults non-stop into the defenders' position. Therefore, success, in this context, is acquisition of the intermediate objective (assault position) with simultaneous infliction of casualties on the defending force. Degrees of success in battle incorporate aspects of number of enemy casualties, number of friendly casualties, and the range between forces at the battle termination if the AP has not been reached.

Preceding the mathematical development of the small unit fire fight model, the dynamics of the encounter are explained. General parameter dependencies, model interactions, and a general scenario are presented in the first section of this chapter to aid organization and coherence of the analytical model. Section B presents the basic assumptions of the model. Then Section C develops the general mathematics of the force attrition rates. The functional forms of the model parameters are presented in Section D, with the final mathematical presentation of the fire fight in Section E.

A. ENGAGEMENT DYNAMICS

The underlying tactical situation of the model is that the engagement constitutes a small unit, daylight attack against an enemy stationary defense. The terrain presents no impassable obstacles to the attacking force, yet provides them with partial defiladed and covered positions when combatants are in prone positions. The tactics employed are either offensive forward fire and movement (toward the defenders) or static position fire (used by both sides). The defending force is assumed static and dug in using available advantages of terrain, cover, and concealment.

Offensive action by the attackers involves two activities: maneuvering and fire support. The individual attacker is performing one of these activities at each moment of the battle, and his assignment to one of the activities depends on the decision of the attacking force commander. All attackers are assigned to units which comprise the attacking force. These units are rotated between the two attacking activities during the forward movement toward the defenders. When attacking combatants are assigned to provide fire support for the maneuvering attackers they comprise the base of fire. Movement of the attacking force is thus incremental.

The maneuvering attackers run without firing toward the defending force a distance determined by tactical policy and then assume a prone firing posture utilizing available cover and concealment from defenders. During this movement the base of fire fires on the defending unit, thus supplying supporting fire for the maneuvering combatants and attriting the enemy. The base of fire fires from the covered and concealed positions which depend upon the terrain and situation, but have no opportunity to dig in or prepare a firing position. When the maneuvering attackers have completed their forward movement, assumed a prone position, and have commenced firing on the defenders, the attackers in the base of fire, or a fraction thereof, move in like manner to an on-line position with the former maneuvering attackers. If only a fraction of the original base of fire now moves in this second movement, the remaining portion plus the original maneuvering attackers now act as a base of fire. This process of fire and maneuver continues incrementally until the entire attacking force is again abreast. At this time fire and movement toward the enemy again commences until the desired assault position is reached. It is noted that movement only to an on-line position is contrary to standard Marine Corps battle battle drill; however, the present assumptions enhance the model's mathematical tractability while not seriously altering the realism of the engagement.

The action of the engagement follows the general tactics presented in Chapter IV, as modified by the above and following assumptions. The battle commences at the LOD when the attacking force sends its first maneuvering combatants forward with simultaneous firing from the base of fire. This offensive action is met by immediate fire from the defending force. Offensive fire and maneuver is carried on until the assault position is reached. This general engagement situation is illustrated by Figure 1.

The complexity of interactions in a combat model yields a difficult web of input parameter dependencies which often become obscured or discarded. [29] However, through careful investigation the major contributing parameters of a combat engagement can explicitly and directly be brought into a descriptive analytic model. As stated in Chapter III, the attrition rate coefficients were considered constant in Lanchester's Square

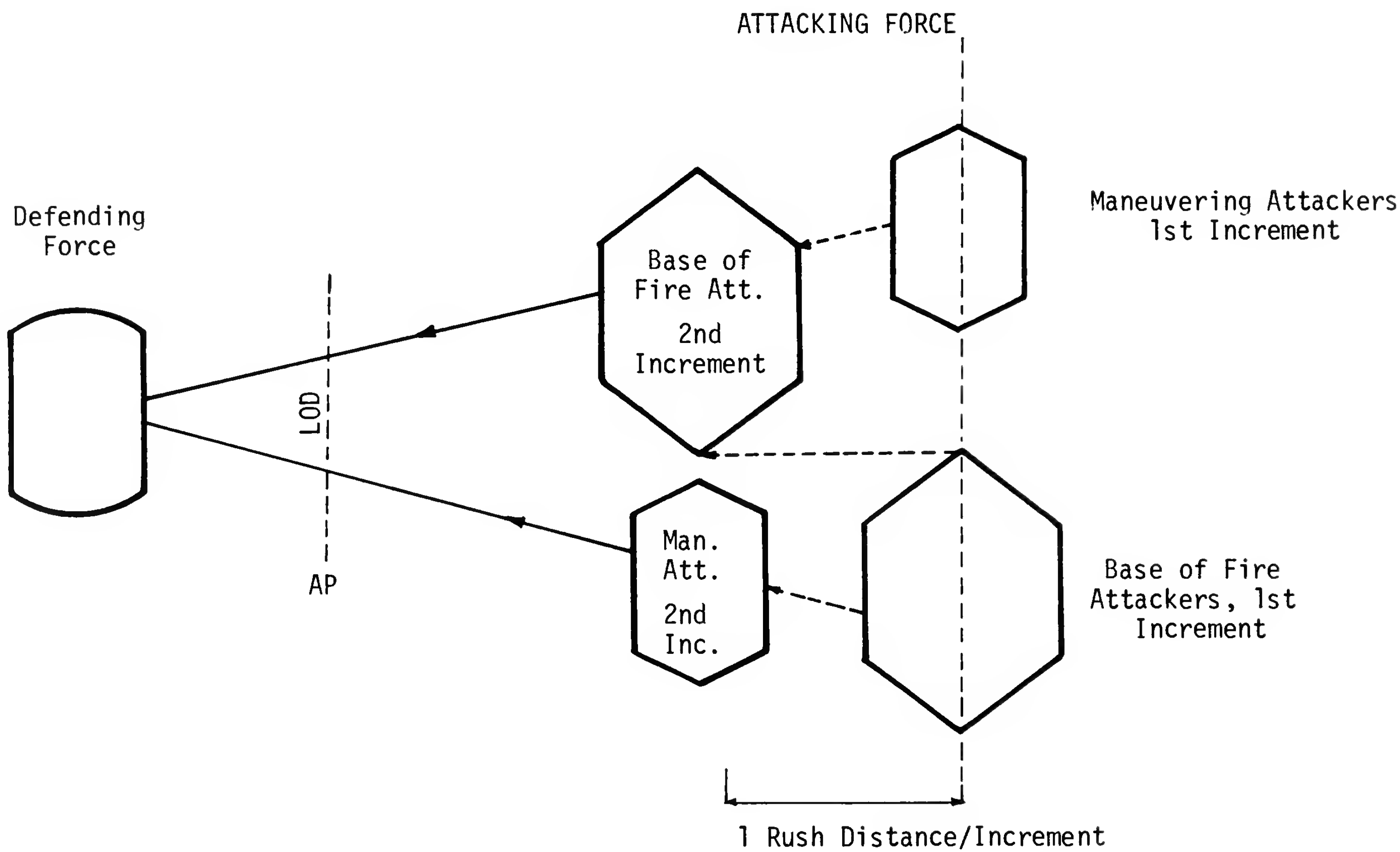

Figure 1
GENERAL ENGAGEMENT SITUATION

Law, yet the realism of combat plainly implies a dependency of this parameter on such quantities as range, battle time, and aiming error. The functional dependencies of attrition rate coefficients open the doors of parameter interactions, but at the same time yield a rich, more rewarding combat model.

As an example of these interactions the rate of advance of a force would appear to depend upon: 1) the number of units comprising the force; 2) tactical policy; and 3) the time differential between incremental movements of the attacking force units. The rate of fire of a force is itself dependent upon force size, available ammunition, range to the target, and speed of advance. Target size and kill rate are directly dependent. Exposure time is functionally related to both rate of advance and accuracy of fire -- which is a factor affecting the probability of kill. Though many human factor variables definitely affect the combat engagement, they present difficulties of explicit analytic expression; thus according themselves more to judgmental insertion into the model through values of input parameters. Hence, most human factor considerations will be implicit in the model. Such implications will be explained as the mathematical model is presented.

B. ASSUMPTIONS OF THE MODEL

The model is continuous in that its time-dependent solutions yield fractional survivors. In actuality casualties are discrete and integer valued; however, the purpose of the model is to reveal tactical implications, not specific numerical casualty predictions. Homogeneity of weapons within and between opposing forces is assumed since the model is cast from an independent small unit engagement unsupported by indirect fire or direct fire of other friendly forces. To gain a factor of conservatism, equality of military abilities between individuals of opposing forces is assumed except for increasing the attackers' aiming error to reflect their energy expenditure during the attack. The general assumptions of fire and movement of the attacking force and the static posture of the defenders have been explained, as were the characteristics of terrain.

Movement of the attacking force is incremental, and the distance moved from prone position to prone position by each unit of the attacking force is termed a rush distance. Tactical policy, dictating these distances, can keep the rushes constant in length over the entire battle period, can functionally vary the rush distances, or can randomly select the incremental distance to move. A portion of the defenders fire at the up and moving attackers while the remainder fire on those attackers comprising the base of fire. During weapon reloading it is assumed that the individual is not exposed to lethal opposing fire, implying less than 100% of a force is available to fire or available as a target to the opposing side.

Fire distribution is assumed to be uniformly distributed over the target area, with the fire being delivered in the aimed mode as opposed to area type fire which generally accompanies indirect fire support weapons or immediate action drills against an ambush. Exposed individuals are assumed to present circular lethal targets; hence, the cumulative circular normal distribution is used to give the probability of impact within the target area. A target kill is implied from a target hit. The above assumptions which are directly related to explicit model parameters will be more fully explained in the following sections.

C. GENERALIZED MODEL

The Lanchester theory presented in Chapter III produced the basic equations of combat:

$$\frac{dA(t)}{dt} = -(p_D)(r_D)[D(t)] \quad , \qquad (1)$$

and

$$\frac{dD(t)}{dt} = -(p_A)(r_A)[A(t)] \quad , \qquad (2)$$

where A(t) and D(t) are the respective force sizes at time t of the attackers and defenders, and $p_D r_D$ and $p_A r_A$ are the attrition rate coefficients with p defined as the probability of kill in an individual encounter and r defined as the rate of encounter. Thus, equation (1) represents the defending force's (D) action against the attacking force (A); and, similarly, equation (2) represents A's action against D. The A force offensive activities are divided into maneuvering and fire support, as previously stated. A1 is defined as the maneuvering combatants and A2 as the base of fire combatants. The size at time t of A1 and A2 is respectively symbolized as A1(t) and A2(t). A1(t) is then the size of A1 at time t and is composed of a command-specified percentage of the entire A force. This percentage, designated as η, is restricted to a value whose inverse yields an integer, and this integer, termed u, implies the total number of units comprising the A force. The number of units in the base of fire is then dependent upon η. Each such unit is symbolized as $A2_i$, $i = 1, \ldots, (u-1)$, and its size at time t is represented as $A2_i(t)$, $i = 1, \ldots, (u-1)$. Therefore the above division of the attacking force is pedantically illustrated by the following equations:

$$A1(t) = (\eta)[A(t)] \quad , \qquad (3)$$

$$u = 1/\eta \quad , \qquad (4)$$

$$A2(t) = \sum_{i=1}^{u-1} A2_i(t) = (1-\eta)[A(t)] \quad , \qquad (5)$$

$$A(t) = A1(t) + \sum_{i=1}^{u-1} A2_i(t) \quad , \qquad (6)$$

There is no requirement that A1 be an entire homogeneous tactical subunit such as one fire team, or one squad. Hence, the A force commander may randomly select individual troops or a unit to comprise A1 for any incremental rush toward the objective.

The defending force is divided such that D1(t) and D2(t) respectively define the number of defenders assigned to fire at the maneuvering attackers and the number of defenders assigned to fire at the A2 base of fire units. The fraction of D comprising D1 is ρ. Therefore,

$$D1(t) = (\rho)[D(t)] \quad , \qquad (7)$$

$$D2(t) = (1-\rho)[D(t)] \quad , \qquad (8)$$

and

$$D(t) = D1(t) + D2(t) \quad , \qquad (9)$$

The need to reload weapons implies that, on the average, a percentage of each force will not be firing at or attriting the opposition, and consequently will not be exposing themselves as possible targets. Let γ be the proportion of a force not firing because of reloading or fire suppression effects. Then, the number of D forces firing at A1 is $(1-\gamma)[D1(t)]$ and the number firing at A2 is $(1-\gamma)[D2(t)]$. Similarly, since it is assumed that A1 does not fire while moving, the number of A forces firing on the defenders is $(1-\gamma)[A2(t)]$ or $(1-\gamma)\sum_{i=1}^{u-1} A2_i(t)$.

Note that,

$$(1-\gamma)[D1(t)] = (1-\gamma)[\rho D(t)] \quad , \qquad (10)$$

$$(1-\gamma)[D2(t)] = (1-\gamma)(1-\rho)[D(t)] \quad , \qquad (11)$$

and

$$(1-\gamma)[A2(t)] = (1-\gamma)(1-\eta)[A(t)] \quad . \qquad (12)$$

Each incremental movement forward in the attack of the entire A force can be envisioned as a time cycle. The length of the cycle is dependent upon the number of units comprising A, the time (TR) it takes one unit to move one rush distance, and the coordinating time (TC) between unit rushes which is necessary for the A force commander to retain control of his force. For example, if $\eta = 1/3$ implying three units of attackers, then a cycle would be illustrated as in Figure 2. Note that the symbols A1 and $A2_i$, $i = 1, \ldots, (u-1)$ are just general designations to describe the A force commander's choice of which combatants to move and which combatants are to act as the base of fire.

Target size depends on the force and the assigned task. Since target area is assumed circular, RAD1 designates the radius of the circular target area equivalent to that presented by an attacker in an up-and-moving posture, RAD2 is the radius of a circular target area equivalent to that presented by a prone attacker, and RAD3 represents the radius of the target area equivalent to that presented by a defender. Since D1 fires at A1, the running unit, it is assumed that they continue to fire at the same A1 individuals as they take cover after completing the rush distance and also during the following period of coordinating time until a new unit assumes the maneuvering task, leaving the base of fire cover and rushing forward. As the newly designated A1 unit commences its rush,

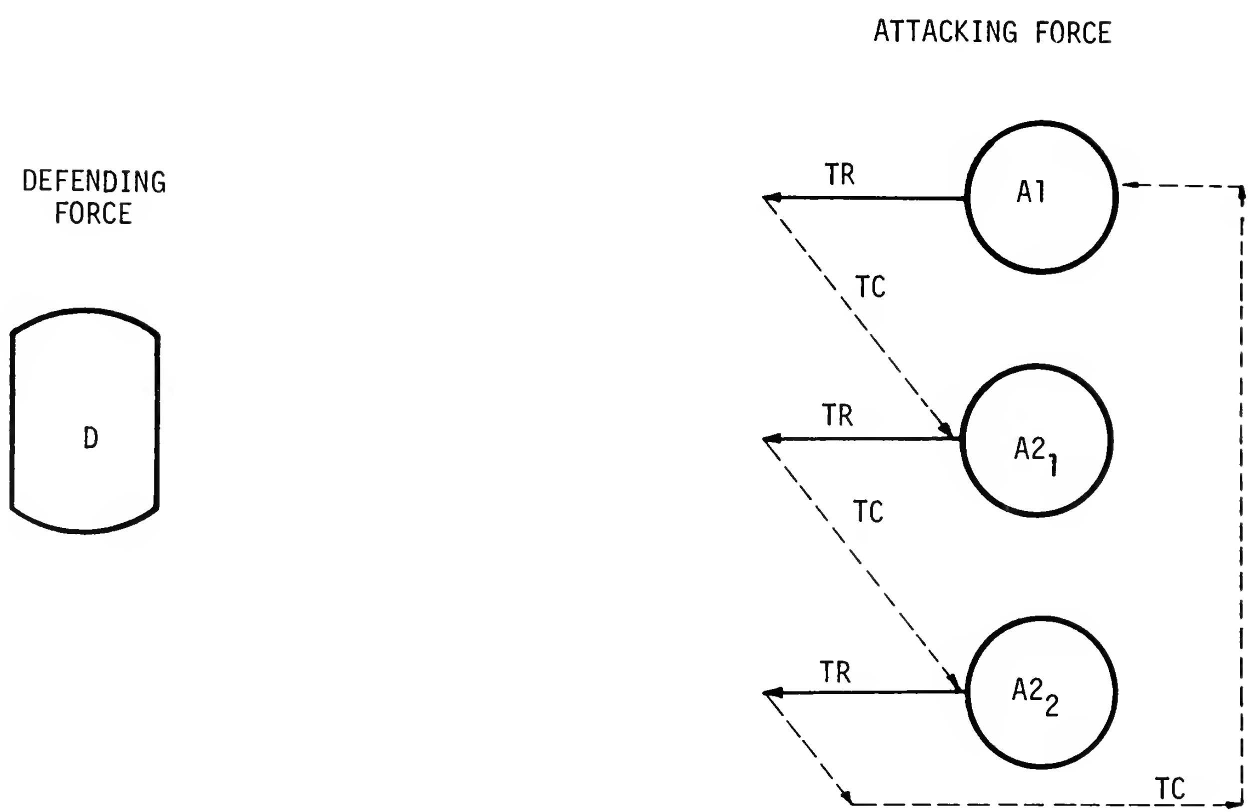

Figure 2
ONE RUSH DISTANCE TIME CYCLE, $\eta = 1/3$,
SHOWING RUSH TIME (TR) AND COORDINATING TIME (TC)

D1 shifts its fire to it. Therefore, in terms of rush times and coordinating times, an A1 unit presents an RAD1 target to D1 for $\frac{TR}{TR + TC}$ per cent of the time and presents an RAD2 target to D1 for $\frac{TC}{TR + TC}$ per cent of the time while it is designated A1. At any instant of time the $A2_i$ unit presents an RAD2 target to D2 for an expected time length of $(i)(TR + TC)$, for $i = 1, \ldots , (u-1)$.

The rate of encounter as appearing in equations (1) and (2) is synonymous with a rate of fire of conventional weapons. For an attacking force it is reasonable to speak of an individual rate of fire since its enemy occupies a static position. For the defending force the reciprocal of rate of encounter yields insight into its process of fire.

Let

$$r_D = 1/T_D \qquad , \qquad (13)$$

and

$$T_D = T_{aD} + T_{fD} \qquad , \qquad (14)$$

where r_D is the rate of fire of the defenders, T_{aD} is the time it takes a defender to search for and acquire a target, and T_{fD} represents the time required by the defender to execute his attack on an A target or, in other words, the time required to fire his weapon. Target acquisition time and firing time are seen to be associated with target cover and concealment. Consequently, an attacker rising from a prone position to move forward in the open causes the defenders' target acquisition time to be negligible. On the other hand, if a combatant remains covered and concealed in his base of fire position his opponents' search time is relatively large in comparison to firing time. Therefore, it is assumed that T_{aD} is approximately zero if the opposing unit is moving in the open, and that T_{fD} is negligible if the attacking force is occupying prone, covered, and concealed positions. The rate of attack of the A force will hereafter be termed rate of fire and will be symbolized by ROF.

Let HPD represent the probability that a D combatant is hit and therefore out of action. The probability that an A1 combatant is killed while running is designated HPA1, and $HPA1_1$ symbolizes the probability of killing an A1 combatant during the TC time units he is prone and getting ready to fire and before another A unit rushes forward. Let $HPA2_i$, $i = 1, \ldots, (u-1)$, represent the probability that a member of the $A2_i$ unit is killed. Sufficient relationships have now been developed to allow a general expression of the model.

The general fire fight is modeled through expression of force loss rates as in equations (1) and (2). Now equation (6) stated

$$A(t) = A1(t) + \sum_{i=1}^{u-1} A2_i(t) \quad ,$$

therefore the attrition rate of A is the sum of the loss rates of its components, or

$$\frac{dA(t)}{dt} = \frac{dA1(t)}{dt} + \frac{d}{dt}\left[\sum_{i=1}^{u-1} A2_i(t)\right] \quad . \qquad (15)$$

In the remaining analytical development, quantities which are time dependent will not be written in functional form unless required for clarity [i.e., A(t) will be written as A].

The D force will not be decomposed into its subunits D1 and D2 for the analytical modeling since all defending forces present similar targets for attrition to the attacking force.

First addressing the rate of encounter parameter, the combination of equation (14) with equation (1) yields

$$\frac{dA}{dt} = -\frac{p_D \cdot D}{T_{aD} + T_{fD}} \quad . \qquad (16)$$

Then by components the A force loss rate becomes

$$\frac{dA1}{dt} = - \frac{p_D \cdot D_1}{T_{fD}} \quad , \qquad (17)$$

and

$$\frac{dA2}{dt} = - \frac{1/(u-1) \sum_{i=1}^{u-1} (p_{D_i})(D2)}{T_{aD}} \quad , \qquad (18)$$

where the factor 1/(u-1) represents the proportion of time that the i^{th} unit of the base of fire presents a prone target to the defenders for (i) · (TR + TC) time units, and where p_{D_i} is the probability of D2 killing an attacker in the $A2_i$ unit. Equations (17) and (18) indicate that target acquisition time for running targets is negligible and firing time against covered targets is not considered in relation to the time it takes to acquire a prone target. The defenders' loss rate becomes

$$\frac{dD}{dt} = -(p_A)(ROF)[(1-\eta)A] \quad , \qquad (19)$$

where it is recalled that $(1-\eta)$ implies only the base of fire combatants attrite D since A1 does not fire while running.

Recalling the relationship of D, D1, and D2, and the reloading time factor, equations (17), (18), and (19) become

$$\frac{dA1}{dt} = \frac{-p_D[(1-\gamma)\rho D]}{T_{fD}} \quad , \qquad (20)$$

$$\frac{dA2}{dt} = - \frac{1/(u-1) \sum_{i=1}^{u-1} p_{D_i} [(1-\gamma)(1-\rho)D]}{T_{aD}} \quad , \qquad (21)$$

and

$$\frac{dD}{dt} = - p_A(ROF)[(1-\gamma)(1-\eta)A] \quad . \qquad (22)$$

Letting γ represent a summation of the fraction of battle time it takes to reload and a fraction incorporating human factor time of not exposing oneself, then $(1-\gamma)$ represents the expected fraction of forces actually available to fire at any instant of time. The model assumes that γ for defenders and attackers is the same.

Substituting, of the fraction of time an A1 combatant presents an RAD1 target with probability of HPA1 of being killed and the fraction of time he is an RAD2 target with $HPA1_1$ probability of being killed, into equation (20) yields

$$\frac{dA1}{dt} = -\left\langle \left[\frac{\frac{TR}{TR + TC}(HPA1)[(1-\gamma)\rho D]}{T_{fD}} + \frac{\frac{TC}{TR + TC}(HPA1)[(1-\gamma)\rho D]}{T_{fD}} \right] \right\rangle \quad . \qquad (23)$$

Since $(1-\gamma)$ also indicates the fraction of firing combatants exposed at any instant of time it thereby decreases the probability of kill, and equations (21) and (22) become

$$\frac{dA2}{dt} = - \frac{(1/(u-1))\sum_{i=1}^{u-1} [(1-\gamma)(HPA2_i)][(1-\gamma)(1-\rho)(D)]}{T_{aD}} \quad , \qquad (24)$$

and

$$\frac{dD}{dt} = - [(1-\gamma)(HPD)](ROF)[(1-\gamma)(1-\eta)A] \quad . \qquad (25)$$

Noting that equations (23) and (24) combine to give dA/dt, then in parametric form the equations (23), (24), and (25) attempt to describe the dynamic small unit engagement.

D. FUNCTIONAL FORMS OF THE MODEL PARAMETERS

As explained at the beginning of this chapter, model interactions and parameter dependencies are numerous. The following sections explain and illustrate some of these interactions and functional forms of the model parameters.

1. Hit/Kill Probabilities

In a combat situation a hit effectively kills an attacking combatant since any wound, lethal or minor, renders the individual's forward movement and further action virtually impossible. A hit on a defender will probably occur in the chest, shoulder, or head area since he is firing from a prone or dug-in position and such a hit effectively implies a kill or serious disablement. This is the basis for the assumption that probability of kill as applied in Lanchester theory equals single-shot hit probability. If the impact point of a rifle round is distributed according to the circular normal distribution with standard deviation σ, then the probability of hitting a circular target of radius r is [2]

$$HP = 1-e^{-\frac{r^2}{2\sigma^2}} \quad . \qquad (26)$$

For aiming error functionally expressed in mils and symbolized as $\alpha(j)$, where j represents target exposure time and for the range between shooter and target expressed as R, the kill probability becomes,

$$HP = 1-e^{-\left[\frac{(30.3)r}{\alpha(j)\cdot R}\right]^2} \quad . \qquad (27)$$

For example, the probability of killing a running A1 target at range R is

$$1-e^{-\left[\frac{(30.3)RAD1}{\alpha(TR)\cdot R}\right]^2} \qquad , \qquad (28)$$

where TR is the time it takes a combatant to move one rush distance or, in other words, the time he exposes himself as an RAD1 target. The other kill probabilities -- $HPA1_1$ and $HPA2_i$ -- are similarly formed.

2. <u>Aiming Error [α(j)]</u>

Aiming error in mils is taken to be dependent upon target exposure time. [22] Therefore, if a combatant is in the base of fire, he presents the defenders with an RAD2 target for $(u-1)\cdot(TR + TC) \cdot (TR + TC)$ time units in each cycle. Aiming error is a monotonically decreasing function for increasing target exposure time with $\alpha(j)$, the attacking combatants' error, being greater than the defenders' aiming error for all j. This disparity of force aiming error is assumed since D is not affected by any exhaustion factor or ability decrement caused by movement. Since A1 combatants are in up and down modes with the same defenders firing at them, an average $\alpha(TR)$, or $\overline{\alpha}(TR)$ where

$$\overline{\alpha}(TR) = [(\eta)\cdot\alpha(\infty) + (1-\eta)\cdot\alpha(TR)] \qquad , \qquad (29)$$

is used for the aiming error of the defenders firing at the A1 combatants.

3. <u>Rates of Fire (^{T}fD, ^{T}aD, ROF)</u>

The D force times for target acquisition and fire are assumed to be linear, monotonic decreasing functions of total battle time shown in Figure 3. The times t_1 and t_2 are respectively the battle times at which the attacking force comes within the maximum effective range of the weapons used by both sides and the time it takes the A force to reach its final assault position. The maximum time to fire is symbolized as TMAXF,

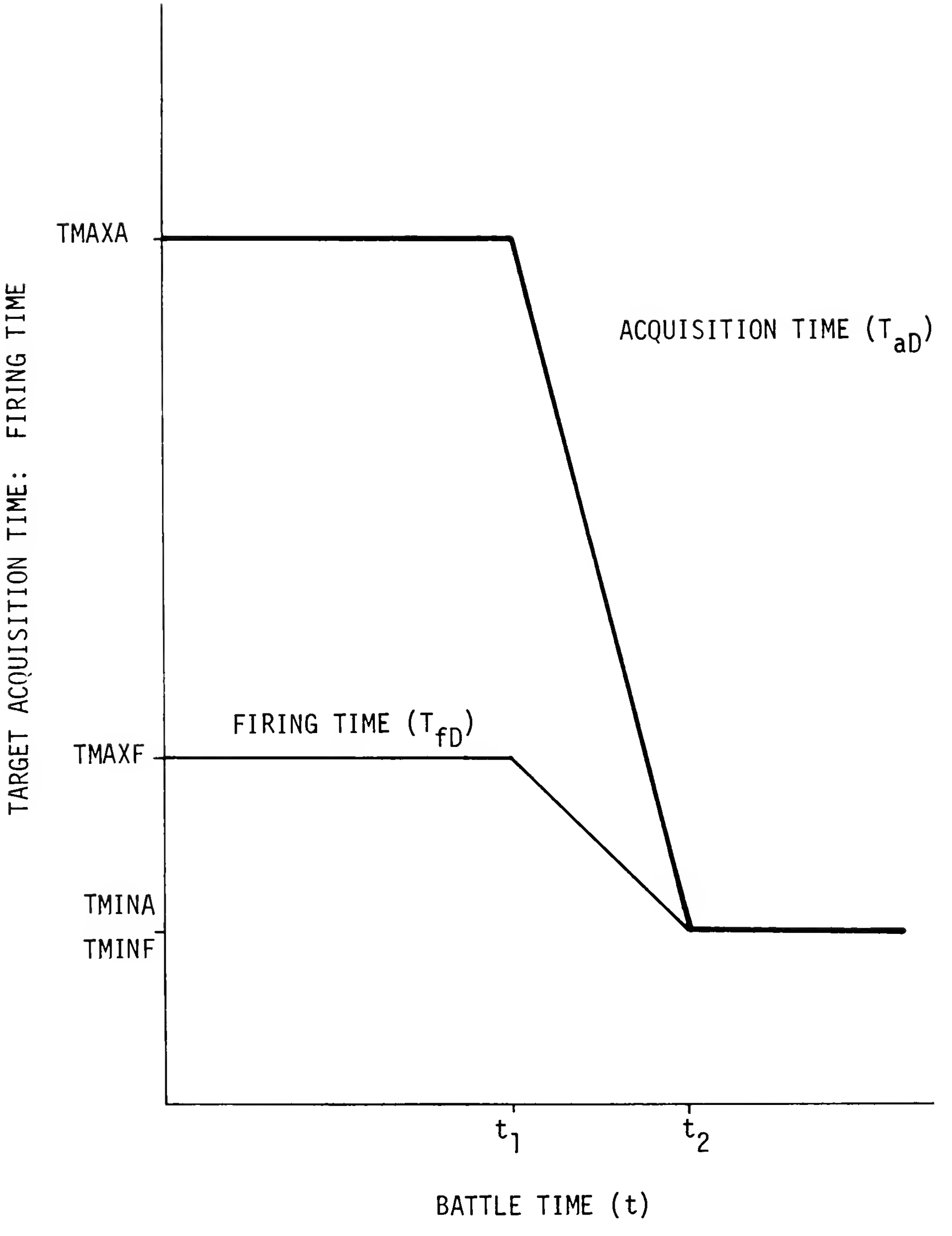

Figure 3
D FORCE TARGET ACQUISITION AND FIRING TIMES

and TMINF represents the minimum time to fire. The maximum time used in acquiring a target is represented by TMAXA, and TMINA is the corresponding minimum time. Therefore,

$$T_{fD}(t) = \begin{cases} \text{TMAXF} & , 0 \le t \le t_1 \\ [\text{TMAXF} - \dfrac{\text{TMAXF} - \text{TMINF}}{t_2 - t_1} \cdot (t - t_1)] & , t_1 \le t \le t_2 \\ \text{TMINF} & , t_2 \le t \end{cases} \quad , \qquad (30)$$

and

$$T_{aD}(t) = \begin{cases} \text{TMAXA} & , 0 \le t \le t_1 \\ [\text{TMAXA} - \dfrac{\text{TMAXA} - \text{TMINA}}{t_2 - t_1} \cdot (t - t_1)] & , t_1 \le t \le t_2 \\ \text{TMINA} & , t_2 \le t \end{cases} \quad . \qquad (31)$$

The rate of fire of the A force is a linearly decreasing function illustrated in Figure 4. The times t_1 and t_2 used in the rate of fire (ROF) formulations correspond to the same values as t_1 and t_2 in the T_{fD} and T_{aD} equations. Hence,

$$\text{ROF} = \begin{cases} \text{ROFMIN} & , 0 \le t \le t_1 \\ [\text{ROFMIN} + \dfrac{\text{ROFMAX} - \text{ROFMIN}}{t_2 - t_1} \cdot (t - t_1)] & , t_1 \le t \le t_2 \\ \text{ROFMAX} & , t_2 \le t \end{cases} \quad , \qquad (32)$$

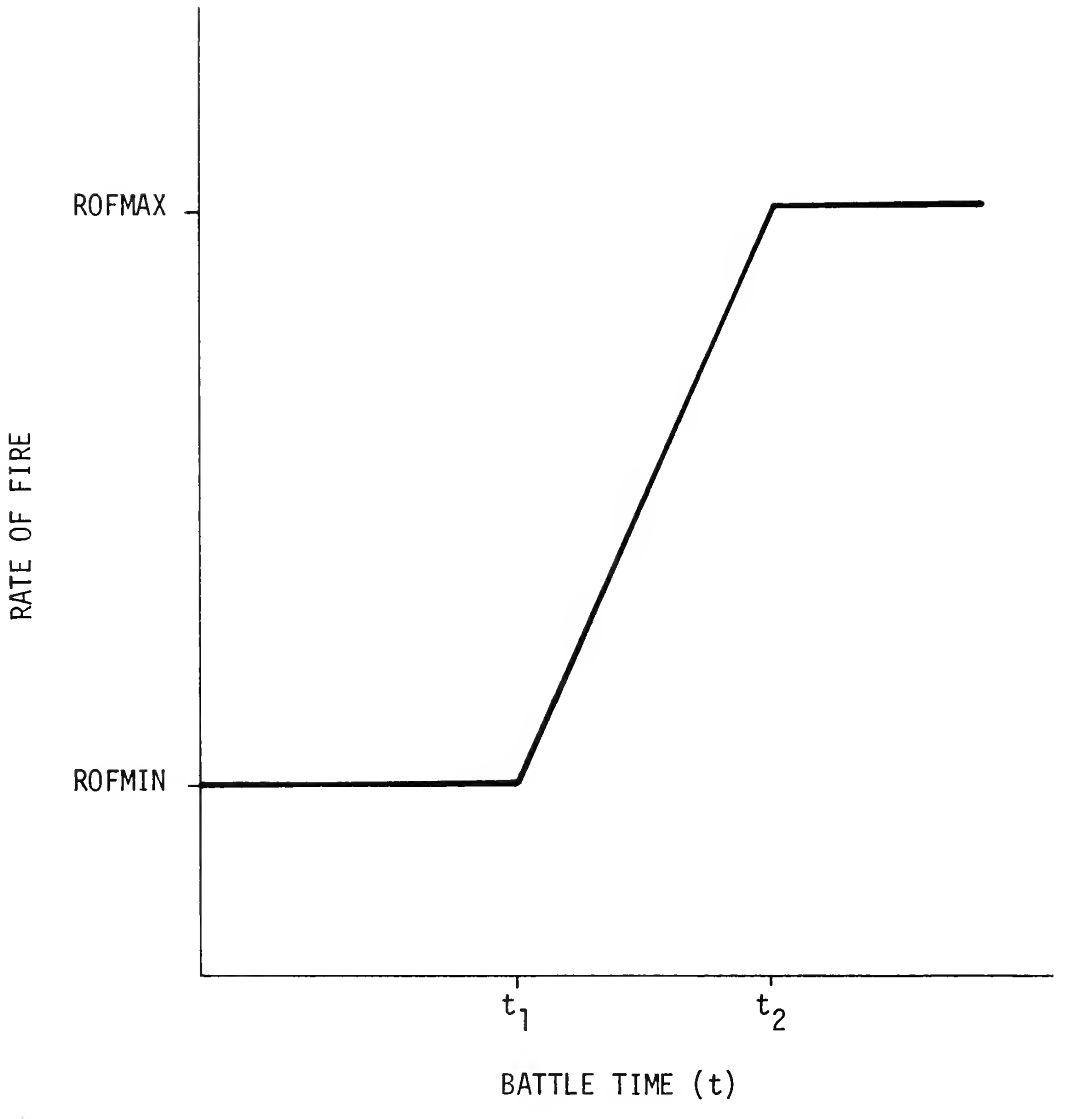

Figure 4
A FORCE RATE OF FIRE

where ROFMAX is the maximum effective rate of fire of the weapon and ROFMIN is the minimum rate of fire. ROFMIN is constrained by

$$\text{ROFMIN} = \begin{cases} \dfrac{2(\lambda)(\text{BL}) - (\text{ROFMAX})(t_2 - t_1)}{t_2 + t_1} & \text{, if the quotient is} > 0 \\ 0 & \text{, if the quotient is} \leq 0 \end{cases} \quad . \qquad (33)$$

The symbols λ and BL are explained in the following paragraph.

4. Ammunition (BL, λ)

The attacking force lethality is constrained by the amount of ammunition it can carry. Basic load of ammunition (BL) is the amount each A combatant can carry. Each attacker is aware he must conserve some ammunition for the final assault from AP into D's position. Let the amount each attacker expends in moving from the LOD to AP be $(\lambda)(\text{BL})$, where $0 \leq \lambda \leq 1$. No constraint is placed on the amount of aummunition available to the defenders since, they are given the capability of previously preparing their defensive posture.

5. Non-Firing/Non-Exposure Factor (γ)

The firing rates, ROF, T_{aD}, and T_{fD}, as presented, do not take into consideration reloading of weapons, battle field confusion, or human factors of battle fatigue or fear. Assuming that a parameter γ exists which accounts for these factors of battle, let it equal the summation of a reloading factor and a fire suppression factor. Let $\gamma_{r\ell}$ equal the fraction of total battle time needed for one combatant to reload, and assume that a combatant does not present a target when reloading. Then $\gamma_{r\ell}$ implies the fraction of a force not exposed at any instant of time.

Let γ_{fs} equal the fraction of A or D not exposing themselves at any instant of time due to fire suppression or other intangible human factor reasons. Then,

$$\gamma = \gamma_{r\ell} + \gamma_{fs} \quad . \qquad (34)$$

6. Rate of Advance (ROA)

A rate of advance (ROA) of the A force is used to calculate t_1 and t_2. This rate is dependent upon tactical policy of movement of the A force, but for any one rush or incremental movement of the entire A force, ROA is directly dependent upon TR and TC, the times needed for one combatant or A1 unit to move one rush distance and the coordinating time between movements of the units of A. Define the time for the entire A force to move one rush distance as,

$$TAR = (u)(TR + TC) \quad . \qquad (35)$$

If the length of a rush distance is symbolized as RD, then

$$ROA = \frac{RD}{TAR} \quad , \qquad (36)$$

in distance per unit time.

7. Range

The range (R) between A and D is as,

$$R(t) = R(0) - (t)(ROA) \quad , \qquad (37)$$

where R(0) is the range at time zero. Though distance between the u units of A may vary due to tactical policy, an averaged range based upon the entire A force rate of advance is used to calculate the force attrition rates.

E. Fire Fight Model

Bringing together the basic analytical developments of the model in Section C and the functional forms of the input parameters of Section D, the final mathematical expression of a small unit engagement with variable attrition rate coefficients can be written. Leaving the A force decomposed in groups A1 and A2 for clarity, the final form of the fire fight model is,

$$\frac{dA1}{dt} = -\left\{\begin{bmatrix} \dfrac{\frac{TR}{TR+TC}\left[1 - e^{-\left[\frac{(30.3)(RAD1)}{\overline{\alpha}(TR)\cdot[R(t)]}\right]^2}\right][(1-\gamma)\rho D]}{T_{fD}} + \end{bmatrix}\right.$$

$$\left.\begin{bmatrix}\dfrac{\frac{TC}{TR+TC}\left[1 - e^{-\left[\frac{(30.3)(RAD2)}{\overline{\alpha}(TR)\cdot[R(t)]}\right]^2}\right][(1-\gamma)\rho D]}{T_{fD}}\end{bmatrix}\right\}, \quad (38)$$

$$\frac{dA2}{dt} = -(1/(u-1))\sum_{i=1}^{u-1}\frac{\left[1-e^{-\left[\frac{(30.3)(RAD2)}{\alpha(i\cdot(TR+TC))[R(t)]}\right]^2}\right][(1-\rho)D](1-\gamma)^2}{T_{aD}}, \quad (39)$$

and

$$\frac{dD}{dt} = -\left[1-e^{-\left[\frac{(30.3)(RAD3)}{[\alpha(\infty)][R(t)]}\right]^2}\right](ROF)[(1-\eta)A](1-\gamma)^2 . \quad (40)$$

It is realized that all the complexities of battle have not been explicitly addressed, and in Chapter VII proposals for possible extensions of this model are presented. The next chapter uses the above equations in an investigation of fire and maneuver policies of ninety different battle simulations, based upon a scenario of a small unit attacking a static defensive force.

VI. INVESTIGATION OF FIRE AND MANEUVER POLICIES BY VARYING FOUR INDEPENDENT VARIABLES

To investigate offensive tactical policies of an attacking infantry unit, a scenario of a small unit daylight attack against a static defensive force was used. The independent variables were force size, rush distance length, the number of units composing the attacking force, and the distribution of the defensive force fire. The combinations of the variables yielded 90 different battle simulations. The following section gives the scenario which is an enhancement of the general model situation presented in Chapter V. Section B of this chapter discusses the nonvariant parameter values of the model, and Section C presents the values of the independent variables which formed the basis of the tactical fire and maneuver policy of each battle simulation. Section D then presents the results of the investigation.

A. SITUATION

The overall engagement situation is that a maneuvering A force attacks a static D force during daylight hours. It is assumed that before the battle begins the A force leader receives an operation order to attack the defended position, has sufficient time to make a reconnaissance of the objective, and selects the line of departure (LOD) and the final assault position (AP). Movement of the A force to the LOD does not involve action from the defender. Once at the LOD, with all attackers in prone positions, the A force leader commences the attack by sending his first maneuvering attackers forward accompanied by simultaneous support from the remaining base of fire. A frontal assault (direct movement toward the enemy) is the basis of the A force tactics. Movement is carried out by the use of constant rush distances and is affected by the number of units comprising the A force. Execution of rushes, defending force posture, and aspects of terrain have been described in Chapter V. With the scenario specified, the values of the model parameters will now be addressed.

B. NONVARIANT PARAMETER VALUES

The following paragraphs give the values of parameters which were held constant during the investigation of tactical policies. The values used for these model parameters were selected from 1) published research and field experiments, 2) military statistics, and 3) personal combat knowledge. Professional military judgment was used to temper any known unrealism of experimental data.

1. Range

The line of departure (LOD) was chosen to be 600 meters from the defenders' position. At this range, or greater, average rifle fire has questionable effect; so, generally, a defending force armed with rifles would not take an opposing force under fire at ranges beyond 600 meters. The final assault position (AP) was 50 meters in front of the D force. This range keeps the A force just beyond hand grenade range of D yet is close enough that the final on-line, non-stop assault through the objective should not falter because of attacker exhaustion.

2. Weapons and Ammunition

As mentioned above, the weapon used by both forces was assumed to be a semi-automatic, .30 calibre, U.S. rifle such as the M-14. The maximum effective range for such a weapon was taken to be 450 meters, where maximum effective range is militarily defined as the range at which a shooter may be expected to fire accurately. [13] However, kills at ranges greater than 450 meters were assumed possible. Although more than twice the load specified by military doctrine, a basic load of ammunition (BL) of 300 rounds per attacker was used. [13] Recent combat experience caused this departure from the written specification. Realizing that conservation of ammunition is a necessity so that the final assault can be carried through, the fraction (λ) of the basic load of ammunition expended by a combatant in the attack, was given a value of 2/3. Although once a force commences an attack it is virtually impossible for the attacking force leader to completely control the force's fire, the fact still remains that each attacker realizes he must conserve some ammunition since resupply in such an attack is impossible. Therefore, setting $\lambda = 2/3$ provides 1/3 of a basic load, or 100 rounds, for the final assault.

3. Target Exposure, Rush, and Coordinating Times

A value of ten seconds was selected for the coordinating time (TC) between rushes to allow the attacking force leader time to decide which combatants to send forward on the next rush. Times longer than ten seconds were not used since they caused the computer battle simulations using a rush distance of five meters to be of excessive length. Corresponding to rush distances of 5, 10, 20, 30, and 40 meters, the target exposure times used were 1.8, 2.8, 4.6, 6.0, and 7.7 seconds. [22] These times correspond to values experimentally obtained by clocking the times needed for a man to rise from a prone position, run the specified distance, and assume another prone position. The times recorded were those clocked between the time the rusher first appeared in view of the observer and the time when the rusher disappeared from view. Because of terrain and foliage these times were slightly shorter than the actual time it took the rusher to move the specified distance. Hence, times to move one rush distance were assumed equal to target exposure times plus one second. This one second addition was assumed to be the time an attacker is moving but not fully seen during each rush.

4. Rates of Fire

The maximum rate of fire of the attacker was taken as 40 rounds per minute, which is the sustained rate of fire of the M-14 rifle. [13] The minimum times to fire and acquire targets by the D force were also taken to correspond to 40 rounds per minute. The maximum time required for a defender to fire his rifle was chosen as 0.059 minutes, which implies 17 rounds per minute. The maximum time for a D force combatant to acquire a target was selected as 0.16 minutes which corresponds to approximately 6 rounds per minute. [10]

5. Aiming Error

Figure 5 shows an assumed relationship between aiming error and target exposure time. The curves are based upon rifle range data reported by T. E. Sterne and K. L. Yudowitch in 1955. [22] The curves were modified to the extent that the defenders' combat, steady-state aiming error is approximately three times greater than the range values. A factor of three seemed reasonably conservative, yet sufficiently real to account for battlefield conditions of excitement and fire suppression. The 10 mil steady-state error for a defender also corresponds to data from rifle range experiments. These experiments found that a 10 mil error corresponds to a poor rifle range shooter [3,23]. A comparison of a poor rifle range shooter to an average combat shooter seemed acceptable. For steady-state aiming error of assaulters, a 20% exhaustion factor was added to the combat steady-state aiming error of the defenders. This implies

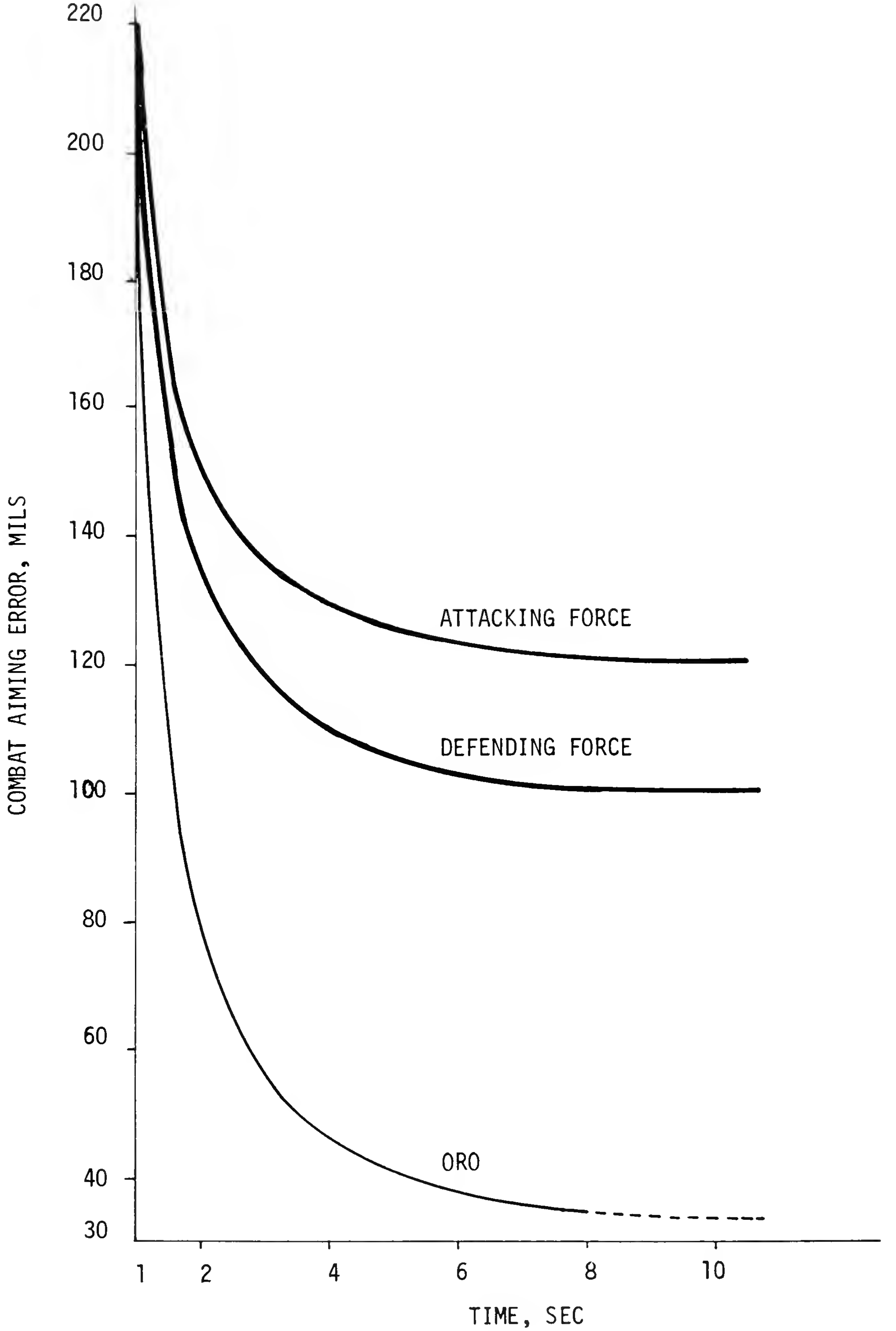

Figure 5
AIMING ERROR vs TARGET EXPOSURE TIME

$\alpha(\infty)$ of attackers equals 12 mils. As the length of target exposure time decreases, rifle range situations come closer to approximating combat situations. Firing at a target exposed for one or one and one-half seconds is basically a reaction instead of a conscious rifle movement and thus there is little difference at such exposure times between rifle range and combat conditions.

6. Reloading and Non-Exposure Factors

Since force rates of fire do not take into consideration reloading of weapons and fire suppression, a factor incorporating these effects was introduced into the model in Chapter V. This factor was symbolized as γ and was set equal to the summation of a reloading factor and a fire suppression factor.

The reloading factor ($\gamma_{r\ell}$) is defined as the fraction of total battle time used by one combatant to reload. Assuming that a combatant is not a visible target when he is reloading, then $\gamma_{r\ell}$ also represents a fraction of the total battle time a shooter is not exposed. A parameter value for $\gamma_{r\ell}$ was obtained from the following assumptions. Let it be assumed it takes six seconds to load a magazine into a rifle [22] and that each combatant has five magazines (one magazine in his rifle and four magazines on his cartridge belt). This implies it will take 24 seconds to load the four remaining magazines into the rifle. Now let 200 available rounds of ammunition [$(\lambda)(BL) = 2/3(300) = (200)$] be distributed such that 20 rounds are in each of the five magazines carried by a combatant (i.e., 100 rounds loaded in magazines) and that 100 rounds are in bandolier ammunition (five rounds per clip). This bandolier ammunition is made so that a magazine can be loaded while still in a weapon, but it is a slower process than just reloading a rifle directly with a full 20-round magazine. Assuming that it takes 20 seconds to fill a magazine using five-round bandolier clips, then loading the 100 rounds of bandolier ammunition into magazines will take 100 seconds. Given each combatant begins the engagement with a full magazine in his rifle, let it be assumed that a combatant will load four magazines into his rifle. When he has expended the rounds from his last magazine, it is assumed he will fill that magazine (the one in the rifle) using five-round bandolier clips. Such a loading procedure of magazines into a rifle and of bandolier ammunition into a magazine (in the rifle) implies a total time of approximately two minutes to reload. Therefore, $\gamma_{r\ell}$ equals the time needed to reload $\lambda \cdot BL$ rounds (i.e., two minutes) divided by battle time (the length of time it takes the A force to move from the LOD to the AP).

The effect of battle field fire suppression is accounted for by a factor symbolized as γ_{fs}. This factor is defined as the fraction of A and D forces not exposing themselves at any instant of time due to fire suppression. A value of 0.15 was chosen for γ_{fs}. This value was selected so that the maximum value of $\gamma = \gamma_{r\ell} + \gamma_{fs}$ would be 0.50. A 0.50 value for the γ factor was placed as an upper limit since it is a close estimate for the reloading factor needed by a fire team with an automatic rifle, where one man of the four must constantly be loading ammunition so that the automatic weapon can be kept firing (Marine Corps doctrine).

7. Target Size

As stated in Chapter V, target size depends on the force and the combatants' assigned tasks. Also it was assumed that circular target area is equivalent to the area presented by a combatant. For a combatant in a prone position and in the attacking force, the radius of the assumed circular target presented by the attacker was taken to be 7.0 inches. [29] For a defender (assumed in a prepared defensive area) a radius of 4.95 inches was used for the radius of a circular target area equivalent to that area presented by a defensive force combatant. [29] For a maneuvering attacker the value 15.2 inches was used for the radius of the circular area equivalent to the area presented by the moving combatant. [7]

All of the above parameter values supply the necessary information to completely evaluate all remaining model variables, except for the independent variables of force size, rush distance, number of units in A, and D force fire distribution which will be specified in the following section.

C. INDEPENDENT VARIABLES OF BATTLE SIMULATIONS

The values of the independent variables used in the battle simulations are given in the following paragraphs. For each battle there was specified a force size for A and D, a number of units composing A, a rush distance, and a D force fire distribution.

1. Rush Distance

The tactical offensive policy used in each computer battle simulation was based on constant rush distance and the number of units comprising the A force. The values used for rush distance were 5, 10, 20, 30, and 40 meters. Rush distance of less than 5 meters was considered impractical since such distances imply an excessively slow rate of advance. Rush distances greater than 40 meters were considered unrealistic in combat because of the weight of equipment carried into battle by each combatant.

2. Number of Units in Attacking Force

The values used for the fraction (η) of A forces assigned to maneuver were 1/5, 1/3, and 1/2. The reciprocal of η is the number of units (u) comprising the attacking force. Therefore the values of u were 5, 3, and 2.

3. <u>Force Size</u>

The attacking force size was set at twelve men for the first 45 battle simulations. Another 45 different battle simulations used a value of 24 for the attacking force size. The force size of the D force was set at four combatants for all engagement simulations.

4. <u>Defensive Fire Distribution</u>

Fire distribution of the defensive force was varied so that the fraction of defenders firing on the maneuvering attackers took on values of 0.1, 0.5, and 0.9. Although values of 0.1, and 0.9 for defensive fire distribution represent extreme situations, such values were chosen so comparison of offensive tactical policies and defensive enemy fire could cover a wide spectrum of possible defensive fire tactics.

D. RESULTS OF INVESTIGATION OF TACTICAL POLICIES

A computer program (in Appendix A) used the parameter values of Section B and the independent variable values of Section C to generate the end of battle data displayed in Tables 1 through 6. Each row in the tables illustrates the data of a specific battle. Difference between battles occurred from varying force size, rush distance (RD), number of units (u) comprising A, and the distribution of D's fire.

The computer program updated battle data every tenth of a minute during the engagement and recorded the status of the battle at every 50 meters of advance of the attacking force. If either force was annihilated before a new 50 meter distance was traversed, the program recorded the battle statistics at the time of annihilation. Given 1) a rush distance,

2) the number of units comprising the A force, and 3) the fire distribution of the defenders, the number of survivors of each force together with the range between forces at the end of the battle and the length of the battle in minutes can be read from the tables on the following pages. The symbols in the column titled VICTOR, in all tables, represent either 1) the force which annihilated its opponents before the attacking force reached the assault position or 2) the force with the greatest number of survivors when the range at the end of the battle was 50 meters (i.e., the attacking force reached the AP).

It is most interesting to note in Tables 1, 2, and 3 that the attacking force was only victorious when the defensive fire was distributed with only one-tenth of the defending force firing at the up and moving attackers ($\rho = 0.1$). Even with this distribution of defending force fire the attackers were not victorious for all tactical policies employed (rush distance combined with the number of units in A). For a rush distance of 5 meters, in combination with all values for the number of units in A, the attacking force was annihilated at or before a range of 100 meters. During such battles, the defenders never lost more than 50% of their own force. For all values of u (number of units in A) when enemy fire was distributed with $\rho = 0.1$, rush distances of 20, 30, and 40 meters yielded higher degrees of battle success (more surviving attackers, fewer surviving defenders, and proximity to the AP) than the shorter RD's of 10 and 5 meters. Against an enemy fire distribution of $\rho = 0.1$ the results showed that 1) a 12-man attacking force of 3 or 5 units should move in 30 and 40 meter rushes and 2) a 12-man attacking force of 2 units could attain approximately the same degree of battle success by rushing in 20 meter increments as they could by employing 30 and 40 meter rush tactics.

TABLE 1

END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.1
AND A FORCE = 12, D FORCE = 4

ETA = 1/5 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	7.22	0	A	85	20.1
30	6.18	0	A	80	24.5
20	2.44	0	A	65	34.9
10	0	2.47	D	115	55.6
5	0	3.34	D	170	94.1

ETA = 1/3 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	8.68	0	A	70	12.4
30	8.24	0	A	75	14.9
20	6.74	0	A	75	20.5
10	0	1.20	D	70	36.7
5	0	2.84	D	125	60.9

ETA = 1/2 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	7.97	1.05	A	50	8.6
30	7.74	0.37	A	50	10.4
20	7.42	0	A	60	14.1
10	1.50	0.38	A	50	25.4
5	0	2.58	D	100	43.0

TABLE 2

END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.5
AND A FORCE = 12, D FORCE = 4

ETA = 1/5 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	C	2.71	D	110	19.0
30	C	2.87	D	125	22.4
20	C	3.18	D	150	29.3
10	C	3.50	D	185	47.7
5	C	3.74	D	225	79.5

ETA = 1/3 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	C	1.70	D	60	12.6
30	C	1.80	D	70	15.0
20	C	2.51	D	100	19.6
10	C	3.10	D	130	32.2
5	C	3.50	D	175	54.5

ETA = 1/2 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	1.10	2.20	D	50	8.6
30	C	2.04	D	50	10.4
20	C	2.11	D	65	13.9
10	C	2.80	D	100	23.0
5	C	3.38	D	140	39.1

TABLE 3
END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.9
AND A FORCE = 12, D FORCE = 4

ETA = 1/5 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	0	3.44	D	160	17.0
30	0	3.51	D	175	20.0
20	0	3.62	D	200	26.0
10	0	3.77	D	230	42.0
5	0	3.88	D	270	70.5

ETA = 1/3 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	0	2.90	D	100	11.6
30	0	3.00	D	115	13.7
20	0	3.28	D	140	17.9
10	0	3.55	D	175	29.3
5	0	3.75	D	215	49.4

ETA = 1/2 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	0	2.83	D	60	8.3
30	0	2.95	D	80	9.8
20	0	3.07	D	100	13.0
10	0	3.40	D	140	21.3
5	0	3.65	D	175	36.0

When the enemy fire distribution factor ρ was increased to 0.5 and to 0.9, the defenders proved to be victorious in all battles against a 12-man attacking force regardless of tactical policies employed by the attackers. Although 12 attackers were never victorious under the increased enemy fire distribution factors, higher degrees of attacking force success were achieved by forces comprised of 2 and 3 units using rush distances of 30 and 40 meters than for other tactical policies. For $\rho = 0.5$ an offensive tactical policy of u equal to 2 and RD equal to 40 proved best when considering the number of attacking survivors. and acquisition of the AP. However, it should be noted that the greatest number of defenders were killed when the tactical policy incorporating u equal to 3 and RD's equal to 30 and 40 meters was employed against an enemy fire distribution of $\rho = 0.5$.

When nine-tenths of the defenders ($\rho = 0.9$) were assigned to fire on the moving attackers of a 12-man attacking force, the defending survivor statistics varied only slightly for all offensive tactical policies. Although defending survivor numbers were generally equal, tactical policies using u equal to 2 and RD equal 30 and 40 allowed the 12-man attacking force to move most of the way to the assault position.

The computer program was run a second time using an attacking force size of 24 men. All other model input parameter values were identical to those in the first program. The data for these battles are presented in Tables 4, 5, and 6. In all battles (except four cases when $\rho = 0.9$) the attacking force was victorious. Where one-tenth of the enemy fire was distributed against the up and moving attackers, the attacking force achieved its greatest success by using tactical policies incorporating u equal to 3 and 5 and RD equal to 30 and 40 meters Success for battles

TABLE 4

END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.1
AND A FORCE = 24, D FORCE = 4

ETA = 1/5 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	21.7	0	A	140	17.9
30	21.3	0	A	140	21.6
20	20.5	0	A	150	29.5
10	18.1	0	A	145	52.4
5	12.7	0	A	130	99.9

ETA = 1/3 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	22.4	0	A	115	11.3
30	22.2	0	A	125	13.5
20	21.6	0	A	130	18.4
10	20.1	0	A	130	32.2
5	16.8	0	A	130	60.2

ETA = 1/2 RHO = 0.1

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	22.4	0	A	65	8.3
30	22.3	0	A	85	9.7
20	22.0	0	A	105	12.9
10	20.7	0	A	110	22.5
5	18.0	0	A	110	41.6

TABLE 5

END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.5
AND A FORCE = 24, D FORCE = 4

ETA = 1/5 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	17.3	0	A	130	18.3
30	16.6	0	A	130	22.1
20	14.7	0	A	130	30.5
10	10.0	0	A	120	55.3
5	0	1.68	D	125	101.4

ETA = 1/3 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	19.5	0	A	110	11.4
30	19.1	0	A	115	13.7
20	17.9	0	A	120	18.7
10	15.3	0	A	120	33.1
5	9.6	0	A	110	63.0

ETA = 1/2 RHO = 0.5

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	19.3	0	A	65	8.3
30	19.4	0	A	80	9.8
20	18.9	0	A	95	13.1
10	16.6	0	A	100	22.9
5	11.7	0	A	95	43.0

TABLE 6

END OF BATTLE RESULTS WHERE
DEFENSIVE FIRE IS DISTRIBUTED AS RHO = 0.9
AND A FORCE = 24, D FORCE = 4

ETA = 1/5 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	11.7	0	A	115	18.9
30	10.3	0	A	110	23.0
20	05.1	0	A	95	32.9
10	0	2.02	D	130	53.8
5	0	2.88	D	175	90.8

ETA = 1/3 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	16.1	0	A	100	11.6
30	15.5	0	A	110	13.9
20	13.5	0	A	110	19.2
10	08.4	0	A	100	34.6
5	0	1.78	D	115	62.1

ETA = 1/2 RHO = 0.9

RD	ATTACK SURVIV	DEFEND SURVIV	VICTOR	RANGE(MET) END BATTLE	BATTLE LENGTH(M)
40	15.6	0	A	60	8.4
30	16.1	0	A	70	10.0
20	15.3	0	A	90	13.3
10	11.3	0	A	90	23.5
5	0	1.18	D	80	44.4

incorporating these tactics was considered greater than battles incorporating other tactics since the enemy was annihilated at greater ranges.

For enemy fire distribution of $\rho = 0.5$ and 0.9, the attackers achieved their greatest success by employing the tactics of u equal to 3 units and RD equal to 30 or 40 meters.

The computer program printed out battle data as shown by the example in Appendix A, and from these results it was noted that single-shot hit probability increased by a factor of approximately 3 for all targets when the range to the target became less than 100 meters. It was also noted that the attacking force rate of fire was extremely low when rush distances of 5 meters were employed. Conclusions concerning the results of the simulations appear in the following chapter.

VII. CONCLUSIONS AND EXTENSIONS

In this chapter general conclusions are made concerning the results presented in Chapter VI. Following the conclusions, suggestions are given for extension and enrichment of the model.

It was noted in Chapter VI that when the twelve-man attacking force used a five meter rush distance they were annihilated. It appears that this type of movement resulted in an excessively slow rate of advance for the attackers, thereby allowing the defenders to expend a great number of rounds in effort to kill the attacking combatants. Also, this slow rate of advance dictated an extremely low rate of fire (less than ten rounds per minute in all cases) for the attackers. The low rate of fire hindered the attackers' destruction of the enemy. It does not seem realistic that a force would continue the attack, taking a great number of casualties, without increasing their rate of fire above the restrictions imposed by the fire fight model assumptions. The attacking force rates of fire, during battles incorporating rush distances greater than five meters, seemed reasonable.

Considering all distributions of enemy fire for the given force sizes, the study suggested that rush distances of 30 and 40 meters were of greater tactical advantage to the attacker than were lesser RD lengths. The only exception to this trend was that the 20 meter rush, in connection with 2 units comprising the A force, produced approximately the same degree of battle success for the attacking force as did the RD's of 30 and 40 meters which were combined with 2 and 3 units in the A force. The results also implied that an A force composed of 2 or 3 units was generally more successful than a 5 unit attacking force. The only exception to this conclusion was a 5 unit A force which employed a rush distance of 40 meters against an enemy fire distribution of $\rho = 0.1$. In this case the attackers achieved approximately the same degree of battle success as did other tactical policy combinations against the same type defensive fires. These general conclusions give support, at the small unit level, to the Marine Corps doctrine of triangular force structure. The results of the study do deviate from Marine Corps battle drill tactics of using rush distance lengths of 5 to 10 meters. [33] Perhaps this deviation of optimal rush length (30 to 40 meters suggested by this study versus 5 to 10 meters suggested by the Marine Corps) stems from the total distance (550 meters) over which the battles in this study were fought. This large variance of results does, however, give reason for continued research and modeling of small unit fire and maneuver tactics.

The development and testing of the fire fight model have indicated areas for future study and analysis which could yield additional insights into the field of small unit tactics. As a first extension of this thesis it is suggested that further application of the model could be made by analyzing additional fire and maneuver policies. The only tactical policy studied in this thesis was that of constant rush distance combined with varying the number of units comprising the attacking force. Other rush distance policies could be analyzed to see which policy yields the greatest degree of success. Additional fire and maneuver tactics which could be studied are 1) rush distances which decrease linearly as the range to the objective decreases, or 2) rush distances which are varied logarithmically with range. Following a logarithmic change of rush distance the attacking force could use either the tactic of long rush distances for the majority of the battle with short rushes used over the last 100 or 200 meters or a policy of beginning with long RD's and then quickly changing to the shorter rush distances for the majority of the advance. Studies incorporating these ideas would entail the modification of the computer program presented in Appendix A but would not require mathematical alteration of the model.

Enhancement of the model analytically can take many forms for it is indeed rare that modeling of human decision processes and combat dynamics could ever be completed. Stamina was not addressed explicitly in model, yet the performance of an attacking combatant decreases steadily in many areas as he continues to move. [16] This energy expenditure factor was implicit in the model since rush distances of greater than 40 meters were not tested. Larger RD's than 40 meters were not used since it was felt that the average combatant, carrying a weapon, body armor, and ammunition, could not continue to rush in increments greater than 40 meters over the entire distance between the LOD and AP. However, direct inclusion of a combatant's performance decrement could enrich the model. The psychological breakpoint of a force [21] could also be included since the number of casualties taken by a force could easily effect its performance. This unit performance variable suggests relations with other parameters such as surrender factors, dessertion rates, and morale coefficients. [23,34] Employment of supporting weapons such as mortars or artillery would be another realistic extension of the fire fight model. In connection with model extensions additional effort might be given to estimation of parameter values. Combat data for parameter estimation is almost impossible to obtain, however if the researcher is not bound by security classification, restricted documents do exist which might aid the modeling of combat situations. [19,30]

Schoderbek suggested that the effect of increased intelligence about an opposing force's position can be simulated by permitting the single-shot kill probabilities to be non-decreasing functions of time. [26] The kill probability functions used in this study incorporated this aspect since they were non-decreasing functions and were dependent upon time. This dependency on time resulted from the kill probabilities being directly dependent on range, where range was dependent upon the rate of advance of the attacking force.

In this thesis the major combat variables have been explored and related in effort to realistically model the small unit infantry battle. Interesting results of the fire fight model application were obtained giving implications that increasing rush distance length might improve success in combat. The author does not imply that present tactics can or should be changed based on the results of this study or other present models of combat engagements. However, it is strongly felt that modeling of military tactics could produce interesting results which might aid the combat commander in his decision-making process. As a final caution to those undertaking research and modeling of military tactics the following quote is offered.

> "If ever there was a world in which situations do not repeat themselves like some mass production model, it is the military world. If we are to avoid the imposition of arbitrary limits to the exercise of judgment and control, let us be careful not to create in a mathematical vacuum situations which are based neither on past experience of affairs, nor on any conception of the innumerable variables and factors that determine social decision either today or tomorrow. The human brain, human values, human judgments, are still superior to the mechanics and processes of electronic computers or guidance systems. The day this ceases to be true there will probably be no human brains. But until then, let us use true scientific method as an aid to human judgment, and not as a hindrance. Science <u>is</u> human experience; it is not an alternative to judgments, and it is certainly not something that can operate outside human experience." [28]

APPENDIX A

Computer Program for a Daylight Attack With Constant Rush Distance

This computer program is written in FORTRAN IV language for the IBM 360 series computer. Preceding the main program is a list of variables and their definitions. Following the main program are six function subprograms. The last two pages of this appendix present sample output data generated by the program for each battle analyzed.

```
C     THIS COMPUTER PROGRAM EVALUATES A DAYLIGHT ASSAULT
C     OF AN INFANTRY RIFLE SQUAD AGAINST AN ENTRENCHED
C     ENEMY FIRE TEAM USING A CONTINUOUS MODEL BASED ON
C     THE LANCHESTER THEORIES OF COMBAT.
C
C
C     THE SYMBOLS USED IN THE PROGRAM ARE DEFINED AS FOLLOWS
C
C     RO = LINE OF DEPARTURE
C     AP = ASSAULT POSITION
C     T = STARTING TIME
C     AO = A FORCE INITIAL SIZE
C     DO = D FORCE INITIAL SIZE
C     ETA = FRACTION OF A FORCE RUNNING
C     RHO = FRACTION OF D FORCE FIRING ON RUNNING A FORCES
C     U = NUMBER OF A FORCE UNITS
C     GAMMA = FRACTION OF COMBATANTS NOT EXPOSED
C             AND NOT FIRING.
C     RD = RUSH DISTANCE
C     TR = TIME PER RUSH DISTANCE PER MAN
C     TE = EXPOSURE TIME
C     TC = COORDINATING TIME
C     TAR = TIME PER RUSH DISTANCE FOR THE A FORCE
C     ROA = RATE OF ADVANCE
C     TMAXF = MAX. TIME TO FIRE, D FORCE
C     TMINF = MIN. TIME TO FIRE, D FORCE
C     TMAXA = MAX.TIME TO ACQUIRE A TGT., D FORCE
C     TMINA = MIN.TIME TO ACQUIRE A TGT., D FORCE
C     ROFMAX = MAX. RATE OF FIRE, A FORCE
C     ROFMIN = MIN. RATE OF FIRE, A FORCE
C     BL = BASIC LOAD OF AMMUNITION
C     LAMDA = FRACTION OF BL USED IN FIRE AND MANEUVER
C     AE1 = AVERAGED AIMING ERROR
C     AE2 = AIMING ERROR(TE + TC)
C     AE3 = AIMING ERROR (2(TE+TC))
C     T1 = TIME FOR A FORCE TO MOVE INTO MAXIMUM EFFECTIVE
C          WEAPON RANGE.
C     T2 = TIME FOR A FORCE TO MOVE TO AP
C
C
C     DATA USED IN GENERATING COMPUTER OUTPUT LOCATED AT END
C     OF THE PROGRAM.
```

MAIN PROGRAM

```
      IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
      REAL*4 RKLDEQ,S
      REAL*8 LAMDA

C     THE FOLLOWING CARDS READ IN THE DATA/VARIABLE
C     CONSTANTS.

      READ(5,101) TMAXA,TMINA,TMAXF,TMINF,RO,AP,BL,LAMDA,
     *AO,DO,ROFMAX
  101 FORMAT(4F15.3)
      DIMENSION RD(5),TR(5),ETA(3),RHO(3),TE(5)

C     RUSH DISTANCE AND THE TIME TO COVER THIS DISTANCE
C     ARE NOW READ INTO THE PROGRAM.  RD = 40,30,20,10,5
C     AND TR = 8.7,7.0,5.6,3.8,2.8 SEC.  .  ALSO TARGET
C     EXPOSURE TIMES ARE READ WHERE TE = 7.7,6.0,4.6,2.8,1.8
```

```
      READ(5,102) (RD(I),I=1,5),(TR(J),J=1,5),(TE(M),M=1,5)
  102 FORMAT(5F12.7)

C     THE VALUES USED FOR ETA = 0.200,0.333,0.5000 AND
C     RHO = 0.100,0.500,0.900 .

      READ(5,103) (ETA(K),K=1,3),(RHO(L),L=1,3)
  103 FORMAT(3F15.8)

      TC = 10.D0/60.D0
      RAD1=15.2D0
      RAD2= 7.0D0
      RAD3= 4.95D0
      DO 10 I=1,5
      TR(I) = TR(I)/60.D0
      TE(I) = TE(I)/60.D0
   10 CONTINUE

C     CALCULATION OF VARIABLES NOT DEPENDENT ON THE
C     INDEPENDENT TIME VARIABLE WHICH IS INCREMENTED
C     DURING THE SOLUTION OF THE DIFFERENTIAL EQUATIONS.
C     THE DO LOOP VARIABLE I CHANGES RUSH DISTANCE (RD)
C     AND RUSH TIME(TR).  THE VARIABLE K ROTATES  THE
C     PARAMETER ETA WHICH IS THE FRACTION OF A FORCES
C     MOVING AT ANY INSTANT OF TIME.

      DO 50 I=1,5
      DO 50 K=1,3
      U =1.D0/ETA(K)
      TAR = U*(TR(I) + TC)
      ROA = RD(I)/TAR
      T1=(RO-450.D0)/ROA
      T2 = (RO - AP)/ROA
      ROFMIN = (2.D0*LAMDA*BL - ROFMAX*(T2-T1))/(T2 + T1)
      IF(ROFMIN) 301,302,302
  301 ROFMIN = 0.D0
  302 CONTINUE
      GRL = 2.D0/((RO - AP)/ROA)
      GAMMA = GRL + 0.15D0
      AE1 = (AIMERR(1000))*ETA(K) + (1.D0-ETA(K))*
     *AIMERR(TE(I))
      AUG2 = TE(I) + TC
      AE2 = AIMERR(AUG2)
      AUG3 = 2.D0*(TE(I) + TC)
      AE3 = AIMERR(AUG3)
      AE4=12.D0

C     THE FOLLOWING CARDS INITIALIZE AND CALL THE RKLDEQ
C     SUBPROGRAM WHICH USES THE RUNGE-KUTTA-GILL FOURTH-
C     ORDER METHOD FOR SOLUTION OF A SYSTEM OF N(IN THIS
C     PROGRAM N=2) FIRST-ORDER ORDINARY DIFFERENTIAL
C     EQUATIONS. THE DO LOOP VARIABLE L CHANGES THE
C     VALUES OF RHO, THE FRACTION OF D FORCES FIRING ON
C     THE UP AND MOVING A FORCES.

      DO 50 L=1,3
      DIMENSION Y(2),F(2)
      Y(1) = A0
      Y(2) = D0
      T = 0.D0
      H = 0.1D0
      NT= 0
      N = 2
      YY = 600.D0

      WRITE(6,201) RO,AP,T,A0,D0,ETA(K),RHO(L),U,GAMMA,RD(I)
```

```
      *,TR(I),TE(I),TC
      *,TAR,ROA,TMAXF,TMINF,TMAXA,TMINA
      *,ROFMAX,ROFMIN,BL,LAMDA,AE1,AE2,AE3,T1

  201 FORMAT('1',//////15X,'INPUT DATA AND COMPUTED',
     1' VARIABLE CONSTANTS'/////15X,'LINE OF DEPARTURE =',
     AF8.1//15X,'ASSAULT POSITION =',F8.1//15X,
     B'STARTING TIME(MIN.) =',
     2F10.8//15X,'A FORCE INITIAL SIZE =',F15.8//15X,
     3'D FORCE INITIAL SIZE =',F15.8//15X,'ETA =',F13.8//15X
     4,'RHO =',F13.8//15X,'NUMBER OF A UNITS =',F6.1//15X,
     5'GAMMA =',F13.8//
     615X,'RUSH DISTANCE =',F13.8//15X,'TIME PER RD PER MAN'
     A,1X,'=',F13.8//15X,'EXPOSURE TIME PER RUSH'
     7,1X,'=',F13.8//15X,'COORDINATION TIME =',F13.8//15X,
     8'TIME PER RD PER A FORCE =',F13.8//15X,'RATE OF',
     9' ADVANCE =',F15.8//15X,'MAX. TIME TO FIRE =',F13.8//
     *15X,'MIN. TIME TO FIRE =',F13.8//15X,'MAX. TIME TO',
     1' ACQUIRE A TGT. =',F13.8//15X,'MIN. TIME TO ACQUIRE',
     2' A TGT.=',F13.8//15X,'MAX.RATE OF FIRE =',F13.8//15X,
     3'MIN.RATE OF FIRE =',F13.3//15X,'BASIC LOAD OF AMMO ='
     4,F6.1//15X,'LAMDA =',F13.8//15X,'AIMING ERROR(AVG)=',
     5F13.8//15X,'AIMING ERROR(TE+TC) =',F13.8//15X,'AIMING'
     6' ERROR(2(TE+TC)) =',F13.8//15X,'T1 =',F15.8)

      WRITE(6,251) T2
  251 FORMAT(//15X,'T2 =',F15.8)

      WRITE(6,205)
  205 FORMAT('1',/////////////)

      WRITE(6,206) RD(I),ETA(K),RHO(L)
  206 FORMAT(35X,'RD =',F5.1,4X,'ETA =',F6.3,4X,'RHO =',F6.3
     *)

      WRITE(6,202)
  202 FORMAT(//10X,'BATTLE',3X,'ATTACK',3X,'DEFEND',3X,'RAN'
     1'GE',4X,'HIT PROB',3X,'HIT PROB',3X,'HIT PROB',3X,
     2'TAD',5X,'TFD',5X,'ROF'/10X,'(MIN.)',3X,'SURVIV',3X,
     3'SURVIV',3X,'TO TGT',3X,'TGT = D',4X,'TGT = A1',3X,
     4'TGT =A23'//)

C     THE FOLLOWING VARIABLES ARE CALCULATED TO SIMPLIFY
C     THE DIFFERENTIAL EQUATIONS.

    1 CONTINUE
      TFDX = TFD(T,T1,T2,TMAXF,TMINF)
      TADX = TAD(T,T1,T2,TMAXA,TMINA)
      RX = R(T,ROA,RO)
      ROFX = ROF(T,T1,T2,ROFMAX,ROFMIN)

C     THE FOLLOWING CARDS ARE THE COMBAT MODEL DIFFERENTIAL
C     EQUATIONS.  Y(1) AND Y(2) ARE THE A FORCE AND D FORCE
C     SIZES RESPECTIVELY AT ANY INSTANT OF TIME(TIME IS
C     INCREMENTED BY 0.1 MIN.) HENCE F(1) AND F(2) REPRESENT
C     THE ATTRITION RATES OF A AND D FORCES RESPECTIVELY.
C     NOTE THAT CARDS 4-7 ARE NOT EXACT RESTATEMENTS OF THE
C     A2 LOSS RATE, HOWEVER SINCE TC = 10 THE APPROXIMATION
C     IS EXACT FOR COMPUTATIONS.

      F(1)=-((TR(I)/(TR(I)+TC))*(1.D0 - DEXP(-((30.3D0*
     1RAD1)/(AE1*RX))**2)) + (TC/(TR(I)+TC))*(1.D0 -
     2DEXP(-((30.3D0*RAD2)/(AE1*RX))**2)))*(1.D0 - GAMMA)*
     3RHO(L)*Y(2)/TFDX  -
     4((1.D0/(U-1.D0))*((1.D0-GAMMA)**2)*(1.D0-RHO(L))*Y(2)
     5*(1.D0-DEXP(-((30.3*RAD2)/(AE2*RX))**2))/TADX)  -
     6((1.D0 - 1.D0/(U-1.D0))*((1.D0-GAMMA)**2)*(1.D0-RHO(L)
     7)*Y(2)*(1.D0-DEXP(-((30.3*RAD2)/(AE3*RX))**2))/TADX)
```

```
      F(2)=-(((1.DO-GAMMA)**2)*ROFX*(1.DO-ETA(K))*(Y(1))*(1.
     1DO -DEXP(-((30.3DO*RAD3)/(AE4*RX))**2)))

      S = RKLDEQ(N,Y,F,T,H,NT)

      IF(S -1.0) 3,1,2

    3 WRITE(6,203)
  203 FORMAT('1',1X///'ERROR STOP')
      STOP

    2 CONTINUE
      HPD  =(1.DO - DEXP(-((30.3DO*RAD3)/(AE4*RX))**2))
      HPA1 =(1.DO - DEXP(-((30.3DO*RAD1)/(AE1*RX))**2))
      HPA23=(1.DO - DEXP(-((30.3DO*RAD2)/(AE3*RX))**2))
      IF((Y(1).LT.0.DO).OR.(Y(2).LT.0.DO)) GO TO 400

C     PRINTED OUTPUT OCCURS AT 50 METER RANGE INCREMENTS. IF
C     EITHER FORCE SIZE IS CALCULATED TO BE LESS THAN OR
C     EQUAL TO ZERO THE BATTLE IS TERMINATED.

      IF(RX.LE.YY) GO TO 400
      GO TO 1

  400 WRITE(6,204) T,Y(1),Y(2),RX,HPD,HPA1,HPA23,TADX,TFDX,
     *ROFX
  204 FORMAT(10X,F6.2,3X,F6.2,3X,F6.2,3X,F5.1,4X,F8.5,3X,
     *F8.5,3X,F8.5,3X,F5.3,3X,F5.3,3X,F4.1)

      IF((Y(1).LT.0.DO).OR.(Y(2).LT.0.DO)) GO TO 50
      YY = YY -50.DO
      IF(RX - 50.DO) 50,50,1
   50 CONTINUE
      STOP
      END
```

FUNCTION SUBPROGRAMS

```
C.........................................................

C         THE FOLLOWING FUNCTION CALCULATES THE RANGE
C         BETWEEN ATTACKER AND DEFENDER.

      REAL FUNCTION R*8(T,ROA,RO)
      IMPLICIT REAL*8(A-H,O-Z)
      R=RO - (T*ROA)
      RETURN
      END


C.........................................................

C         THE FOLLOWING FUNCTION PRODUCES THE TIME D FORCE
C         COMBATANTS REQUIRE TO FIRE THEIR WEAPONS.

      REAL FUNCTION TFD*8(T,T1,T2,TMAXF,TMINF)
      IMPLICIT REAL*8(A-H,O-Z)
      IF(T.LT.T1) GO TO 1
      IF(T.GE.T1.AND.T.LT.T2) GO TO 2
      TFD = TMINF
      GO TO 3
    1 TFD = TMAXF
      GO TO 3
    2 TFD = TMAXF - ((TMAXF - TMINF)/(T2-T1))*(T-T1)
    3 RETURN
      END


C.........................................................

C         THE FOLLOWING FUNCTION YIELDS TARGET ACQUISITION
C         TIME FOR D FORCES.

      REAL FUNCTION TAD*8(T,T1,T2,TMAXA,TMINA)
      IMPLICIT REAL*8(A-H,O-Z)
      IF(T.LT.T1) GO TO 1
      IF(T.GE.T1.AND.T.LT.T2) GO TO 2
      TAD = TMINA
      GO TO 3
    1 TAD = TMAXA
      GO TO 3
    2 TAD = TMAXA-((TMAXA -TMINA)/(T2 - T1))*(T-T1)
    3 RETURN
      END
```

```
C.....................................................

C         THE FOLLOWING FUNCTION COMPUTES THE RATE OF FIRE
C         OF THE A FORCES(ROUNDS PER MIN.)

      REAL FUNCTION ROF*8(T,T1,T2,ROFMAX,ROFMIN)
      IMPLICIT REAL*8(A-H,O-Z)
      IF(T.LT.T1) GO TO 1
      IF((T.LT.T2).AND.(T.GE.T1)) GO TO 2
      IF(T.GT.T2) GO TO 3
    1 ROF = ROFMIN
      GO TO 4
    2 TX = T2 -T1
      IF(TX.GT.10.D0) GO TO 10
      GO TO 22
   10 RF = 400.D0/TX
      ROF = RF*(T - T1)/(T2 - T1)
      GO TO 4
   22 ROF = ROFMIN + (ROFMAX - ROFMIN)*(T-T1)/(T2-T1)
      GO TO 4
    3 ROF = ROFMAX
    4 RETURN
      END

C.....................................................

C         THE FOLLOWING FUNCTION YIELDS THE VALUE OF RIFLE
C         AIMING ERROR/RADIAL DISPERSION OF ROUNDS(MILS).

      REAL FUNCTION AIMERR*8(AUG)
      IMPLICIT REAL*8(A-H,O-Z)
      X = 7.5D0/60.D0
      X1=1.85D0/60.D0
      X11=1.75D0/60.D0
      X2=2.85D0/60.D0
      X21=2.75D0/60.D0
      X3=4.65D0/60.D0
      X31=4.55D0/60.D0
      X5 =6.05D0/60.D0
      X51=5.95D0/60.D0
      IF(AUG.GE.X) GO TO 1
      IF((AUG.LE.X1).AND.(AUG.GE.X11)) GO TO 2
      IF((AUG.LE.X2).AND.(AUG.GE.X21)) GO TO 3
      IF((AUG.LE.X3).AND.(AUG.GE.X31)) GO TO 4
      IF((AUG.LE.X5).AND.(AUG.GE.X51)) GO TO 6
    1 AIMERR = 10.D0
      GO TO 7
    2 AIMERR = 14.D0
      GO TO 7
    3 AIMERR = 12.D0
      GO TO 7
    4 AIMERR = 10.7D0
      GO TO 7
    6 AIMERR = 10.25D0
      CONTINUE
    7 RETURN
      END
```

```
C.............................................................
C          FORTRAN 4 VERSION OF RUNGE-KUTTA-GILL ROUTINE


      FUNCTION RKLDEQ (N,Y,F,X,H,NT)
      REAL*8 Y,F,X,H,Q,H1,H2,H3,H6
      DIMENSION Y(2), F(2), Q(25)
C
      NT = NT +1
      GO TO (1,2,3,4),NT
    1 H1 = H
      H2 = H1 * .5D0
      H3 = H1 * 2.D0
      H6 = H1/6.D0
      DO 11 J =1,N
   11 Q(J) = 0.D0
      A = .5D0
      X = X + H2
      GO TO 5
C
    2 A = .2928932188134525
      GO TO 5
C
    3 A = 1.7071067811865475
      X = X + H2
      GO TO 5
C
    4 DO 41 I = 1,N
   41 Y(I) = Y(I) + H6 * F(I) -Q(I)/3.D0
      NT = 0
      RKLDEQ =2.
      GO TO 6
C
    5 DO 51 L = 1,N
      Y(L) = Y(L) + A *(H * F(L) -Q(L))
   51 Q(L) = H3 * A *F(L) +(1.D0-3.D0*A) *Q(L)
      RKLDEQ =1.
C
    6 RETURN
      END
```

INPUT DATA AND COMPUTED VARIABLE CONSTANTS

```
LINE OF DEPARTURE =   600.0
ASSAULT POSITION =    50.0
STARTING TIME(MIN.) =0.0
A FORCE INITIAL SIZE =    12.00000000
D FORCE INITIAL SIZE =     4.00000000
ETA =    0.50000000
RHO =    0.10000000
NUMBER OF A UNITS =   2.0
GAMMA =    0.38334954
RUSH DISTANCE =  40.00000000
TIME PER RD PER MAN =   0.14500000
EXPOSURE TIME PER RUSH =   0.12833333
COORDINATION TIME =   0.16666667
TIME PER RD PER A FORCE =   0.62333333
RATE OF ADVANCE =    64.17112299
MAX. TIME TO FIRE =   0.05900000
MIN. TIME TO FIRE =   0.02500000
MAX. TIME TO ACQUIRE A TGT. =   0.16000000
MIN. TIME TO ACQUIRE A TGT.=   0.02500000
MAX.RATE OF FIRE =  40.00000000
MIN.RATE OF FIRE =       13.812
BASIC LOAD OF AMMO = 300.0
LAMDA =    0.66666667
AIMING ERROR(AVG)=  10.00000000
AIMING ERROR(TE+TC) =  10.00000000
AIMING ERROR(2(TE+TC)) =  10.00000000
T1 =      2.33750000

T2 =      8.57083333
```

RD = 40.0 ETA = 0.500 RHO = 0.100

BATTLE (MIN.)	ATTACK SURVIV	DEFEND SURVIV	RANGE TO TGT	HIT PROB TGT = D	HIT PROB TGT = A1	HIT PROB TGT =A23	TAD	TFD	ROF
0.10	12.00	4.00	593.6	0.00044	0.00600	0.00128	0.160	0.059	13.8
0.80	11.98	3.99	548.7	0.00052	0.00702	0.00149	0.160	0.059	13.8
1.60	11.95	3.97	497.3	0.00063	0.00854	0.00182	0.160	0.059	13.8
2.40	11.92	3.96	446.0	0.00079	0.01061	0.00226	0.159	0.059	14.1
3.20	11.88	3.93	394.7	0.00100	0.01353	0.00288	0.141	0.054	17.4
3.90	11.83	3.90	349.7	0.00128	0.01719	0.00367	0.126	0.050	20.4
4.70	11.74	3.84	298.4	0.00175	0.02354	0.00504	0.109	0.046	23.7
5.50	11.61	3.74	247.1	0.00256	0.03415	0.00734	0.092	0.042	27.1
6.30	11.38	3.58	195.7	0.00407	0.05387	0.01167	0.074	0.037	30.5
7.10	10.95	3.28	144.4	0.00747	0.09674	0.02135	0.057	0.033	33.8
7.80	10.15	2.74	99.5	0.01567	0.19298	0.04445	0.042	0.029	36.8
8.60	7.97	1.05	48.1	0.06522	0.59978	0.17652	0.025	0.025	40.0

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A Lanchester Based Model for Analyzing Infantry Fire and Maneuver Tactics

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis; October 1969

5. AUTHOR(S) *(First name, middle initial, last name)*

Donald Edward Christy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1969	79	34

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

In an effort to analyze small unit fire and maneuver tactics, Lanchester's Square Law is used as the basis for a model relating major combat variables of infantry engagements. An investigation encompassing 90 different computer battle simulations with varying levels of attacking force size, rush distance length, number of units composing the attacking force, and defending force fire distribution is reported. Conclusions of the results of the battle simulations and suggested extensions of the model are discussed.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Lanchester Equations						
Small-unit, Infantry Tactics						
Combat Modeling						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

A-31409